BASIC RELATIONSHIPS BETWEEN HYDROOPTICAL CHARACTERISTICS

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16. Abstract

Definitions are presented of the basic light field parameters in the sea and apparent hydrooptical characteristics. Based on the radiation transfer equation for the two-flux approximation, relationships are considered between inherent and apparent hydrooptical parameters.

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Parameters of the Light Field in the Sea and Hydrooptical Characteristics /5*

The study of the relationships between various hydrooptical characteristics is of considerable interest not only because it makes it possible to determine certain parameters through others, but mainly because they can be used as a convenient tool for analyzing the structure of the light field in the sea. For this purpose it is particularly important to consider the relationships between the inherent, or "primary" hydrooptical characteristics, dependent solely on the properties of a medium, and the "secondary" characteristics which are governed both by the properties of a medium and the nature of a radiation, i.e., its direction and angular distribution.

The theoretical bases of the relationships between the various optical characteristics in the sea are based on the radiation transfer equation. It should be pointed out at the very beginning, however, that the majority of the relationships in which we are interested can be obtained from this equation which describes the case of propagation of radiation in a plane-parallel medium in the one-dimensional or so-called "two-flux" approximation. In this case, as far as the sea is concerned, we shall be considering the "descending" radiation flux (we shall represent it by $\downarrow$) propagating vertically into the depths of the sea from the upper hemisphere into the lower, i.e., extended in a solid angle of two π steradians, whose axis is directed at the nadir, and the "ascending" flux (represented by $\uparrow$), running in the opposite direction. The initial analysis of the light field structure in the atmosphere using this method was performed by Schuster [1]. It must be taken into account that the use of the two-flux approximation, generally speaking, may lead to significant errors whose determination, incidentally, was performed in a paper by Zege [2]. However, the use of this approximation in problems involving the optics of the sea is sufficiently accurate due to the low accuracy of measurement of hydrooptical

*Numbers in the margin indicate pagination in the foreign text.

characteristics, so that resorting to more accurate but much more complex solutions is far from always advantageous at the present time. The use of this approximation in the optics of the sea is also valuable because the solution of many problems in applied hydrophotometry requires a knowledge of precisely the descending or ascending fluxes or their ratio. Many authors [3-9] have used the two-flux method in solving various problems involving optics of the sea. In addition, several empirical relationships have been established between the various hydrooptical characteristics; these relationships were found both in laboratory conditions and during experimental work at sea. Particular attention attaches to the relationships between these characteristics at considerable optical depth, where, as we know from the theory and numerous experiments, a certain quasistationary distribution of brightness by angles is established. The shape of the body of distribution of brightness in this case is governed only by the optical characteristics in the given region of the ocean or sea and is independent of both the angular parameters of the sources creating the light field and the depth. In our literature, it has become conventional to refer to these conditions as the "depth mode"[1]. The radiation transfer equation in this case is considerably simplified and its solutions make it possible to find the optical characteristics on the basis of comparatively simple measured parameters of the light field in the sea. /6

The processes of absorption and scattering of radiation propagated in the sea cannot be evaluated quantitatively by means of classical primary optical characteristics — the index of absorption χ, scattering σ and attenuation ε and the indicatrix of scattering. Their use is valid only for the case of passage of a practically parallel beam of monochromatic radiation through a small homogeneous volume of sea water. The conditions for the propagation of a directed-diffuse or diffuse radiation with complicated angular distribution can be evaluated only by means of certain secondary hydrooptical characteristics which can therefore be interpreted as "characteristics of the light field in the sea."

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[1]Works by foreign authors in this case usually use the terminology "asymptotic distribution of brightness", or "state of optical equilibrium".

The complete representation of the structure of the field formed by non-polarized light can give the angular distribution of radiation brightness at various depths. However, it is necessary to keep in mind at all times that measurement of underwater brightness of radiation in the sea poses an extremely difficult and frequently simply impossible problem at various zenith distances and at various azimuths[2]. This fact makes it necessary to seek out other and /7 more simple measurable parameters which, though they may not be as detailed, are still sufficiently accurate in their characterization of the basic outlines of the structure of the light field in the sea. It is very useful in this connection to use the one-dimensional approximation mentioned above.

As the basic original values, let us use the irradiation of the horizontal surface by the descending flux $E_\downarrow$ (illumination from above) and ascending flux $E_\uparrow$ (irradiation from below), the spatial irradiation E and half-spatial irradiation in the upper hemisphere $E_\downarrow$ and lower hemisphere $E_\uparrow$ (Figure 1). Let us use B (z, Θ, φ) to represent the brightness of the radiation traveling in the sea at a depth z in a direction determined by angles Θ and φ (Θ is the polar angle calculated from the vertical axis directed into the depths of the sea and φ is the azimuth). Then the irradiation of the horizontal surface located in the sea at a depth z will be:

a) descending flux

$$E_\downarrow(z)=\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} B(z,\theta,\varphi)\sin\theta\cos\theta\, d\theta; \tag{1}$$

b) ascending flux

$$E_\uparrow(z)=\int_0^{2\pi} d\varphi \int_{\frac{\pi}{2}}^{\pi} B(z,\theta,\varphi)\sin\theta\cos\theta\, d\theta. \tag{2}$$

The difference between irradiations produced by descending and ascending /8 fluxes on opposite surfaces of a horizontal area may be referred to as the radiation flux density for this area

$$E(z)=E_\downarrow(z)-E_\uparrow(z) \tag{3}$$

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[2]The performance of such measurements requires highly accurate stabilization of the sensors, which can be achieved for example by using gyroscopic devices.

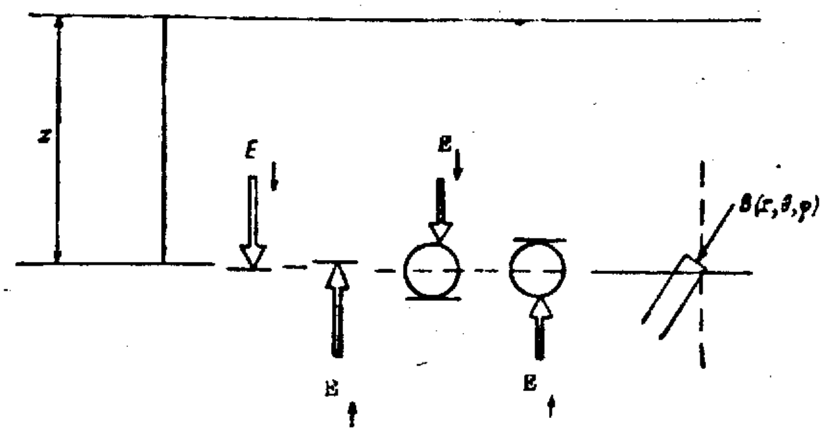

Figure 1. Diagram of Measurements of Basic Radiation Field Parameters.

From the spatial irradiation

$$E(z)=\int_0^{2\pi} d\varphi \int_0^{\pi} B(z,\ \theta,\ \varphi)\sin\theta\, d\theta \tag{4}$$

we can form the following derived values:

a) half-space irradiation by descending flux:

$$E_{\downarrow}(z)=\int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} B(z,\ \theta,\ \varphi)\sin\theta\, d\theta; \tag{5}$$

b) half-space irradiation by ascending flux:

$$E_{\uparrow}(z)=\int_0^{2\pi} d\varphi \int_{\frac{\pi}{2}}^{\pi} B(z,\ \theta,\ \varphi)\sin\theta\, d\theta. \tag{6}$$

The coefficients of angular distribution of radiation, characterizing its direction in the descending, ascending and total radiation fluxes, are very important parameters in the light field in the sea. Let us say that radiation whose brightness is $B(z,\ \Theta,\ \varphi)$ strikes a certain plane layer dz which is located in the sea at a depth z. This radiation will lead to irradiation from above and below ($E_{\downarrow}$ and $E_{\uparrow}$), spatial irradiation E and half-spatial irradiations $E_{\downarrow}$ and $E_{\uparrow}$. The value which is the inverse of the average (weighted arithmetic average of the brightness) cosine $\bar{\mu}$ of the angles of inclination of light beams within the limits of the solid angle of 4 π steradians

$$\bar{\mu}(z) = \frac{\int_{4\pi} \mu B(z, \theta, \varphi)\, d\omega}{\int_{4\pi} B(z, \theta, \varphi)\, d\omega},$$

(where μ equals $\cos\Theta$) will be **referred** to as the coefficient of angular distribution of radiation for the total flux

$$\bar{g}(z) = \frac{1}{\bar{\mu}(z)} = \frac{E(z)}{E(z)} = \frac{E(z)}{E_{\downarrow}(z) - E_{\uparrow}(z)} \tag{7}$$

In the corresponding fashion we can obtain the coefficients of angular distribution for the descending $\mu_{\downarrow}$ and ascending $\mu_{\uparrow}$ fluxes: /9

$$g_{\downarrow}(z) = \frac{1}{\bar{\mu}_{\downarrow}(z)} = \frac{E_{\downarrow}(z)}{E_{\downarrow}(z)};\quad g_{\uparrow}(z) = \frac{1}{\mu_{\uparrow}(z)} = \frac{E_{\uparrow}(z)}{E_{\uparrow}(z)}. \tag{8}$$

If a nearly parallel beam of light is perpendicularly incident on a layer dz, $g_{\downarrow}(z) = 1$. For the same beam, incident at an angle Θ_0 to the vertical, $g_{\downarrow}(z) = \sec\Theta_0$. If the layer is illuminated by isotropic radiation, $g_{\downarrow}(z) = 2$.

In a great many problems involving the optics of the sea, it is necessary to consider the propagation of daylight in it, and this is not only complex (non-monochromatic) radiation, but non-directional radiation, usually composed of direct rays from the Sun, diffuse light from the sky and light which is scattered in the surface layers of the sea, i.e., in other words, the radiation is directionally diffuse (mixed). Sometimes this radiation is completely diffuse, for example, at great optical depths. It is also necessary to deal with mixed radiation when using artificial light sources.

We have already mentioned above that in order to have a quantitative estimate of the propagation conditions of such radiation it is necessary to resort to secondary optical characteristics. Their most complete description can be found in a work by Preisendorfer [7]. On the other hand, the list of secondary characteristics enumerated in this article may be supplemented by several very important values, and on the other hand some of the parameters that are given can be discarded without any damage to the theoretical constructions and practical calculations. For example, there is hardly any need for operating with such values as the indices of attenuation of half-spatial irradiation. At the same time, however, this work does not reflect such important characteristics as the coefficient of underwater irradiation and the coefficient of brightness of the depth of the sea. The latter value is worthy of special attention inasmuch as it makes it possible to estimate objectively the color of the sea and to calculate the masking coefficients in various regions of the oceans and seas. This coefficient, which is very easy to obtain by means of measurements even from an aircraft [10], and possibly from artificial Earth satellites in the near future, is closely linked to other optical characteristics [11]. It is not always possible to agree with the terminology proposed by Preisendorfer. Therefore, it is necessary to unify the concepts of the secondary hydrooptical characteristics, combining them into a simple system and giving them determinations and designations in accordance with the standards adopted in the USSR.

a. Indices of Attenuation of Irradiation. It is well known from numerous experiments that natural light[3], propagating in the sea, is attenuated with depth according to the exponential law, but with an index of attenuation which differs from that of directional radiation ε, which comes under the Buger law. Let us refer to this index of descending flux as $\alpha_{\downarrow}$. Then, using the consept of the irradiation from above on a layer which is located at a depth z in the sea, we will have /10

$$E_{\downarrow}(z)=E_{\downarrow}(0)\,e^{-\alpha_{\downarrow}(z)\,z}. \tag{9}$$

The index $\alpha_{\downarrow}$ has been used very widely in optical studies in the sea and is usually referred to as the index of vertical attenuation. It is measured, as a rule, for separate layers of water $(z_2 - z_1)$:

$$E_{\downarrow}(z_2)=E_{\downarrow}(z_1)\,e^{-\alpha_{\downarrow}(z)\,(z_2-z_1)};\qquad z=\frac{z_1+z_2}{2}. \tag{10}$$

Changing to the differential form, we will have

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[3]Here and in the rest of the paper, the term "natural light" will be understood to be not unpolarized light, as is usually done in physical optics, but "astronomical" light, i.e., light from the Sun, sky, Moon, etc., in contrast to light from artificial radiation sources.

$$\alpha_{\downarrow}(z) = -\frac{1}{E_{\downarrow}(z)} \cdot \frac{dE_{\downarrow}(z)}{dz}. \tag{11}$$

Experimental data indicate that as one would expect from the theory, the index of vertical attenuation is not a constant, even in homogeneous waters, but changes with increasing depth, tending toward some limiting value $\alpha(z)_{z\to\infty} = \gamma$, which is reached when the depth mode is achieved.

In a manner which is completely analogous to that used for the index of vertical attenuation, for descending flux, one can determine the same index for the ascending flux:

$$\alpha_{\uparrow}(z) = -\frac{1}{E_{\uparrow}(z)} \cdot \frac{dE_{\uparrow}(z)}{dz}. \tag{12}$$

Attenuation of natural light with depth is a very convenient method of characterizing the coefficient of underwater irradiation η, which is understood to be the ratio of the irradiation of the descending flux of the horizontal surface located in the sea at a depth z to the simultaneous value of irradiation $E_{\downarrow}(0)$ of a horizontal plane located immediately below the surface of the sea, i.e., illuminated by radiation which has already undergone reflection and refraction at the air-water interface: /11

$$\eta(z) = \frac{E_{\downarrow}(z)}{E_{\downarrow}(0)}. \tag{13}$$

The index of attenuation of spatial irradiation χ, by analogy with (11), is determined as follows:

$$\chi(z) = -\frac{1}{E(z)} \cdot \frac{dE(z)}{dz}. \tag{14}$$

b. Coefficient of Diffuse Reflection of the Depth of the Sea R(z). This is the ratio of the irradiation of a horizontal surface located in the sea at a certain depth z, ascending flux, to the irradiation of the same surface by the descending flux

$$R(z) = \frac{E_{\uparrow}(z)}{E_{\downarrow}(z)}. \tag{15}$$

c. Coefficient of Brightness of the Depth of the Sea $\rho(\Theta, \varphi)$ — this is the ratio of the brightness of the diffuse radiation emerging from the depths of the sea in a direction which is governed by angles Θ and φ (Θ is the zenith distance, φ is the azimuth, calculated from the Sun's vertical) to the brightness of an ideal scatterer B_0 under the same illumination conditions:

$$\rho(\theta, \varphi) = \frac{B(\theta, \varphi)}{B_0}. \tag{16}$$

Usually the coefficient of brightness of the depth of the sea is measured at the nadir, i.e., for angle $\Theta = 0°$

$$\rho(0) = \frac{B(0)}{B_0}.$$

The dependence of the brightness coefficient of the depth of the sea on the wavelength of the light characterizes the distribution of the energy in the radiation spectrum coming from the depths of the sea and consequently its inherent light.[4]

The depth of visibility z_δ (relative transparency) is the depth at which a standard white disc disappears (Sekki disc), thrown overboard on the shady side of the ship (determined visually).

Effective Hydrooptical Characteristics. The absorbing and scattering properties of water in a given region of the ocean or sea for real light fluxes (natural light, artificial sources — floodlights, lasers, etc.) may be estimated /12 quantitatively by means of a group of secondary hydrooptical characteristics which take into account the angular distribution of the incident radiation. We shall refer to these characteristics as effective and, in contrast to the primary characteristics (used for the passage of a practically parallel beam of light through small homogeneous volumes of water) — the indices of absorption χ, scattering σ and attenuation ε — we shall refer to them as χ_{ef}, σ_{ef} and ε_{ef}.

Let us specify a given horizontal elementary layer dz in the sea at a depth z (Figure 2). Let us refer to the brightness of the radiation incident at a solid angle $d\omega$ on this layer from above by $B(z, \Theta, \varphi)$. Then the flux of the

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[4]In contrast to the visible light from the sea, dependent upon the state of its surface, the nature of its illumination and the visual angle of the observer, the inherent light from the sea is determined only by processes of absorption and scattering taking place in its depths.

radiation which will be incident per unit of the surface of the layer in question will be

$$\Phi_{\downarrow}(z)=\int_{2\pi} B(z,\ \theta,\ \varphi)\cos\theta\, d\omega.$$

On passing through the layer dz a portion of this flux $d\Phi_{\chi\downarrow}$ will be absorbed

$$d\Phi_{\varkappa\downarrow}(z)=-\varkappa(z)\,dl\int_{2\pi} B(z,\ \theta,\ \varphi)\cos\theta\, d\omega,$$

where

$$dl=\frac{dz}{\cos\theta} \quad \text{and}$$

$$\frac{d\Phi_{\varkappa\downarrow}}{\Phi_{\downarrow}(z)}=-\varkappa(z)\,dz\,\frac{\int_{2\pi} B(z,\ \theta,\ \varphi)\, d\omega}{\int_{2\pi} B(z,\ \theta,\ \varphi)\cos\theta\, d\omega}. \tag{17}$$

As we can see, the ratio of the integrals on the right hand side of this equation constitutes the coefficient of the angular distribution of the radiation in the descending flux $g_{\downarrow}(z)$ (8). Finally, we will have

$$d\Phi_{\varkappa\downarrow}=-g_{\downarrow}(z)\,\varkappa(z)\,\Phi_{\downarrow}(z)\,dz.$$

The coefficient of proportionality $g_{\downarrow}(z)\cdot\chi(z)$, which is a quantitative estimate of the absorption by a single volume of water of the radiation which is incident on the layer of water from above, will be referred to as the effective index of absorption for the descending flux

$$\varkappa_{\Phi\downarrow}(z)=g_{\downarrow}(z)\,\varkappa(z). \tag{18}$$

Inasmuch as coefficient $g_{\downarrow}(z) \geqslant 1$, the effective index of absorption is always greater (or equal, with normal incidence of parallel rays on the layer) than the index of absorption χ.

In the same way, we can obtain the effective index of attenuation for the ascending flux:

$$\varkappa_{\Phi\uparrow}(z)=g_{\uparrow}(z)\,\varkappa(z). \tag{19}$$

Using a method similar to that for the effective index of absorption, we can introduce the concept of the effective index of scattering. It must be mentioned at the outset, however, that we shall be interested only in the backscattering, since the light which is scattered forward will blend with the /13 total radiant flux passing through layer dz. Let us represent the fraction of the descending light flux scattered backward by N and the ascending flux by M. Dimensionless coefficients N and M will be determined as properties of the medium (for nonpolarized light, by the indicatrix of scattering), and by the angular distribution of the radiation in the incidence flux [12]. Then the effective index of scattering for the descending flux will be

$$\sigma_{\mathrm{eff}\downarrow}(z)=g_{\downarrow}(z)\,N(z)\,\sigma(z), \tag{20}$$

and for the ascending flux

$$\sigma_{\mathrm{eff}\uparrow}(z)=g_{\uparrow}(z)\,M(z)\,\sigma(z). \tag{21}$$

Some idea of the order of magnitude of the coefficient N can be given by its values as obtained by O. V. Kopelevich from measurements of brightness carried out by D. Tyler [13]. The calculations were performed on the basis of the indicatrix of scattering measured by various investigators and averaged by N. Yerlov (Table 1).

The values of N are listed in Table 2.

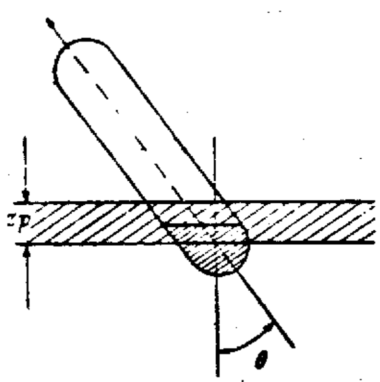

Figure 2. Passage of Radiation Through An Elementary Layer of Water.

The effective indices of attenuation for the descending and ascending fluxes are expressed respectively as follows: /14

$$\varepsilon_{\mathrm{ef}\downarrow} = \chi_{\mathrm{ef}\downarrow} + \sigma_{\mathrm{ef}\downarrow} = g_{\downarrow}(\chi + N\sigma); \tag{22}$$

$$\varepsilon_{\mathrm{ef}\uparrow} = \chi_{\mathrm{ef}\uparrow} + \sigma_{\mathrm{ef}\uparrow} = g_{\uparrow}(\chi + M\sigma). \tag{23}$$

TABLE 1.

Degrees	[illegible]	Degrees	[illegible]	Degrees	[illegible]	Degrees	[illegible]
1	11 000	20	62	75	1.5	130	0.9
5	1 200	30	21	90	1.0	150	1.3
10	280	45	6.7	105	0.8	165	1.8
15	120	60	2.8	120	0.8	180	2.5

TABLE 2.

Observation conditions	Depth, m	N
Funny weather (height of the Sun $h_{\odot} = 45°$)	4.2	1.35
	10.4	1.48
	16.6	1.53
	29.0	1.67
	41.3	1.67
	53.7	1.67
	61.1	1.67
Cloudy sky	6.1	1.65
	18.3	1.67
	30.5	1.70
	55.0	1.76
		1.79

When introducing the concepts of the effective hydrooptical characteristics, no assumptions were made relative to the directionality of the incident radiation, keeping in mind that in the general case this radiation may contain both directional and diffuse components. It will become clear from the discussion as it proceeds that in order to have a more complete analysis of the structure of the light field in the sea, in many instances these components must be separated. A. A. Gershun [4] mentioned not only the desirability but also the necessity of such a differentiation.

Note that all of the secondary characteristics are easily determined experimentally. In order to measure them, in addition to the standard hydrooptical apparatus, it is necessary to have only an underwater photometer with an attachment for measuring the spatial and half-spatial irradiation.

Relationships Between Hydrooptical Characteristics

In conjunction with the fact that we shall be using comparatively many different hydrooptical characteristics in the rest of this presentation, let us explain their designations first:

$\varkappa$ - index of absorption of directional radiation;

δ - index of forward scattering of directional radiation;

β - index of backscattering of the directional radiation;

σ - total index of scattering of directional radiation;

ε - index of attenuation of directional radiation;

$\alpha_{\downarrow\uparrow}$ - index of vertical attenuation of descending (ascending) flux;

χ - index of attenuation of spatial irradiation;

R - coefficient of diffuse reflection of the depth of the sea;

ρ - coefficient of brightness of the depth of the sea;

z_{δ} - depth of visibility (relative transparency);

$\chi_{ef_{\downarrow\uparrow}}$ - effective index of absorption of descending (ascending) flux;

$\varepsilon_{ef_{\downarrow\uparrow}}$ - effective index of attenuation of descending (ascending) flux;

a - index of absorption of diffuse radiation;

$\beta_{ef_{\downarrow\uparrow}}$ - effective index of backscattering of descending (ascending) flux;[5] /15

b - index of back scattering of diffuse radiation;

c - index of attenuation of diffuse radiation;

γ - index of attenuation in the depth mode.

The relationships between the primary hydrooptical characteristics are very simple and are determined by the following relationships:

$$\varkappa+\sigma=\varepsilon; \tag{24}$$

$$\sigma=\lambda+\beta; \tag{25}$$

$$\Lambda=\frac{\sigma}{\varkappa+\sigma}. \tag{26}$$

Λ is the survival parameter of the photon;

$$\zeta=\frac{\varkappa}{\sigma}. \tag{27}$$

ζ is the specific absorbing capacity of the medium.

The relationships between the primary and secondary hydrooptical characteristics, like those between the secondary characteristics themselves, are quite diverse. Let us begin by examining these relationships in the general case, with propagation of a directional-diffuse form of radiation in the sea (for example, daylight or radiation from artificial sources), and then deal in detail specifically with the relationships between the optical characteristics in the conditions of the depth mode, i.e., with completely diffuse radiation.

Let us turn first of all to the radiation transfer equation for two-flux approximation. The descending flux $\Phi_{\downarrow}$ will be attenuated as the result of the

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[5]Formally this index was represented by σ_{ef}. In the following, by analogy with the index of back scattering of directional radiation, we shall represent it as β_{ef}.

true absorption $\chi_{ef_\downarrow}\Phi_\downarrow$ and back scattering $\beta_{ef_\downarrow}\Phi_\downarrow$ and will be simultaneously reinforced thanks to the back scattering of the ascending flux $-\beta_{ef_\uparrow}\Phi_\uparrow$. As we have already mentioned earlier, the forward-scattered light blends with the descending flux without causing its amplification. Similar relationships can be found regarding the ascending flux. Then the transfer equations in the differential form will have the following appearance:

$$\frac{d\Phi_\downarrow(z)}{dz} = -\left[\varkappa_{э\Phi_\downarrow}(z) + \beta_{э\Phi_\downarrow}(z)\right]\Phi_\downarrow(z) - \beta_{э\Phi_\uparrow}(z)\Phi_\uparrow(z);$$
$$-\frac{d\Phi_\uparrow(z)}{dz} = -\left[\varkappa_{э\Phi_\uparrow}(z) - \beta_{э\Phi_\uparrow}(z)\right]\Phi_\uparrow(z) + \beta_{э\Phi_\downarrow}(z)\Phi_\downarrow(z) \qquad (28)$$

These equations are more exact than the classical equations for two-flux approximation according to Schuster, in which the coefficients for $\Phi_\downarrow$ and $\Phi_\uparrow$ are assumed to be the same. /16

Dividing the first equation in system (28) into $\Phi_\downarrow(z)$ and the second into $\Phi_\uparrow(z)$ we will have

$$\alpha_\downarrow(z) = \varkappa_{э\Phi_\downarrow}(z) + \beta_{э\Phi_\downarrow}(z) - \beta_{э\Phi_\uparrow}(z)R(z); \qquad (29)$$

$$-\alpha_\uparrow(z) = \varkappa_{э\Phi_\uparrow}(z) + \beta_{э\Phi_\uparrow}(z) - \beta_{э\Phi_\downarrow}(z)\cdot\frac{1}{R(z)}. \qquad (30)$$

It is clear from equations (29) and (30) how the indices of attenuation of irradiation (indices of vertical attenuation) $\alpha_\downarrow(z)$ — (11) and $\alpha_\uparrow(z)$ — (12) are related to the absorbent and scattering properties of sea water. Solving equation (29) for R(z), the coefficient of diffuse reflection of the sea (15), we will have

$$R(z) = \frac{\varkappa_{э\Phi_\downarrow}(z) + \beta_{э\Phi_\downarrow}(z) - \alpha_\downarrow(z)}{\beta_{э\Phi_\uparrow}(z)}. \qquad (31)$$

From (29) and (30) we can find [7] a still simpler expression for R(z) which is of considerable interest for practical calculations inasmuch as it links directly the coefficient of diffuse reflection of the sea with the indices of vertical attenuation for the ascending and descending fluxes and the true absorption:

$$R(z) = \frac{\alpha_\downarrow(z) - \varkappa_{э\Phi_\downarrow}(z)}{\alpha_\uparrow(z) + \varkappa_{э\Phi_\uparrow}(z)}. \qquad (32)$$

Using the concepts of the coefficients of angular distribution of radiation for total (7) and descending and ascending (8) we can obtain a very important and interesting relationship for R(z) showing that this parameter is completely determined by the coefficients just mentioned, i.e., essentially by the structure of the light field in the sea:

$$R(z)=\frac{\bar{g}(z)-g_{\downarrow}(z)}{\bar{g}(z)+g_{\uparrow}(z)}. \tag{33}$$

Let us compare the latter equation with (31). Taking into account the determination of the index of vertical attenuation $\alpha_{\downarrow}(z)$ through other optical characteristics (29), as well as the small size of $\beta_{ef\uparrow}(z)$, relationship (33) which has been found makes it possible to draw an important conclusion which indicates that the structure of the light field in the sea, generally speaking, is determined by the scattering of the descending flux backward and the true absorption.

For a further consideration of the relationships between the various optical characteristics, following the methods used by Gamburtsev [3], Joseph [5], Duntly [14] and others, we shall divide the descending flux $\Phi_{\downarrow}$ into the directional component $\Phi_{\downarrow}'$ and diffuse component $\Phi_{\downarrow}''$: /17

$$\Phi_{\downarrow}(z)=\Phi_{\downarrow}'(z)+\Phi_{\downarrow}''(z).$$

Now we can write once more the equations for radiation transfer in differential form, but separately for the direct and diffuse radiation. In the most complete form the equations of this system must be written as follows (for a sufficiently deep sea, where it is possible to disregard the light reflected from the bottom):

$$\frac{d\Phi_{\downarrow}''(z)}{dz}=-\varkappa_{\Phi_\downarrow}\Phi''(z)-\beta_{\text{э}\Phi_\downarrow}\Phi_{\downarrow}''(z)+\beta_{\text{э}\Phi_\uparrow}\Phi_{\uparrow}(z)+\delta\Phi_{\downarrow}'(z); \tag{34}$$

$$-\frac{d\Phi_{\uparrow}(z)}{dz}=-\varkappa_{\Phi_\uparrow}\Phi_{\uparrow}(z)-\beta_{\text{э}\Phi_\uparrow}\Phi_{\uparrow}(z)+\beta_{\text{э}\Phi_\downarrow}\Phi''(z)+\beta\Phi_{\downarrow}'(z); \tag{35}$$

$$\frac{d\Phi_{\downarrow}'(z)}{dz}=-(\varkappa+\delta+\beta)\Phi_{\downarrow}'(z)=-\varepsilon\Phi_{\downarrow}'(z). \tag{36}$$

The solution of this system is quite tedious and it is a difficult matter to use it for engineering calculations. Therefore, taking into account the comparatively small difference in the indices of absorption and back scattering for the descending and ascending fluxes, and also taking into account the low accuracy of the hydrophotometric measurements in the sea, we can assume that the effective indices of absorption and back scattering are equal for both fluxes. The solutions that are obtained in this case provide completely satisfactory accuracy and are convenient for carrying out practical calculations. A comparison of the data obtained with the assumption that $\chi_{ef_\downarrow} = \chi_{ef_\uparrow}$ and $\beta_{ef_\downarrow} = \beta_{ef_\uparrow}$, with the data from experimental measurements carried out with different heights of the Sun even at low optical depths ($\tau < 2.5$) has shown that the difference between the results is 7 to 10% [9].

Let us write these equations in the following form (equation (36), we shall assume, does not undergo any changes and is presented here only for the generality of the system):

$$\begin{aligned} \frac{d\Phi''_\downarrow(z)}{dz} &= -a\Phi''_\downarrow(z) - b\Phi''_\downarrow(z) + b\Phi_\uparrow(z) + \beta\Phi'_\downarrow(z); \\ -\frac{d\Phi_\uparrow(z)}{dz} &= -a\Phi_\uparrow(z) - b\Phi_\uparrow(z) + b\Phi''_\downarrow(z) + \beta\Phi'_\downarrow(z); \\ \frac{d\Phi'_\downarrow(z)}{dz} &= -(\varkappa + \beta + \gamma)\Phi'_\downarrow(z) = -\varepsilon\Phi'_\downarrow(z) \end{aligned} \tag{37}$$

Let $\Phi'_\downarrow(0) = \Phi'_{\downarrow 0}$; $\Phi''_\downarrow(0) = \Phi''_{\downarrow 0}\Phi_\downarrow(0) = \Phi_0$. According to the determination /18

$\Phi_\uparrow(0) = R\Phi_\downarrow(0)$.

Integration of the third equation in system (37) gives the familiar Buger law:

$$\Phi'_\downarrow(z) = \Phi'_0 e^{-\varepsilon z}. \tag{38}$$

Considering that within the limits of a given layer the coefficients are constant with $\Phi''_\downarrow(z)$, $\Phi''_\downarrow(z)$ and $\Phi_\uparrow(z)$, the solution of system (37) can be found in the form [5]

$$\begin{aligned} \Phi_\downarrow(z) &= \left[\Phi_0 + n_1\Phi'_0\left(1 - e^{-(a-\varepsilon)z}\right)\right] e^{-\varepsilon z}; \\ \Phi_\uparrow(z) &= \left[\Phi_0 + n_1\Phi'_0\left(\frac{n_2}{n_1} \cdot \frac{\varepsilon + a}{\varepsilon - a} e^{-(a-\varepsilon)z}\right)\right] \frac{\varepsilon - a}{\varepsilon + a} e^{-\varepsilon z}. \end{aligned} \tag{39}$$

Here

$$n_1 = \frac{(a+\varepsilon)(\Delta a + \Delta b) + b\,\Delta a}{\varepsilon^2 - c^2}; \qquad n_2 = \frac{\alpha\,\Delta b + \beta\,\Delta a}{\varepsilon^2 - c^2};$$
$$\varepsilon = \varkappa + \sigma = \varkappa + \delta + \beta; \quad \Delta a = a - \varkappa; \quad \Delta b = b - \beta;$$
$$c = \sqrt{a^2 + 2ab}. \tag{40}$$

Equations (39) clearly show that the Buger law in its simple form (38) cannot be used for the flux of directional-diffuse radiation. By introducing the concept of the index of vertical attenuation, expression (11), this law was formally extended to the flux of mixed radiation. If we solve equation (11) relative to $\alpha_{\downarrow}(z)$ and substitute into it the values $\Phi_{\downarrow}(z_1)$ and $\Phi_{\downarrow}(z_2)$ from the the first equation in system (39), then if we let

$$e^{-(\varepsilon - c) z_1} = A \text{ and } e^{-(\varepsilon - c) z_2} = B,$$

we will have

$$\alpha_{\downarrow}(z) = c - \frac{1}{z} \ln \left\{ 1 - \frac{n_1 \Phi_0'}{\Phi_0} (A + B) \left[1 - \frac{n_1}{\Phi_0} (1 - B) + \right.\right.$$
$$\left.\left. + \frac{n_1 \Phi_0'}{\Phi_0} (1 - B^2) \right] \right\}. \tag{41}$$

Hence, even in regions with homogeneous waters, the index of vertical attenuation, strictly speaking, is not a constant, but approaches some boundary value c with increasing depth.

With $\Phi_0' = 0$ we will obtain the relationships for purely diffuse radiation:

$$\Phi_{\downarrow}(z) \text{ [illegible]}; \quad \Phi_{\uparrow}(z) = \text{[illegible]} \frac{a}{c + a} e^{\text{[illegible]}} \tag{42}$$

These equations are similar in form to Buger's law. It is clear from them that the parameter c, regardless of how it was introduced and regardless of how it was determined in (40) is the index of attenuation of diffuse radiation. /19

An expression very similar in form to (40) was obtained in [9] in establishing the relationship between the index of vertical attenuation and the primary hydrooptical characteristics

$$\alpha_{\downarrow}(z) = g_{\downarrow}(z) \sqrt{\varkappa^2 + 2\varkappa\beta}. \tag{43}$$

The experimental tests of this relationship yielded completely satisfactory results [9].

From system (39) we can obtain still another relationship for determining the coefficient of diffuse reflection of the sea R:

$$R=\frac{c-a}{c+a}\left[1+\frac{1-\frac{n_2}{n_1}\cdot\frac{c+a}{c-a}}{(1+S)\,e^{(\varepsilon-c)z}-1}\right]. \tag{44}$$

where $S=\frac{\Phi_0}{n_1\Phi'_0}$.

In [11], the correlation relationship between the index of vertical attenuation $\alpha_{\downarrow}$ and the spectral distribution of the coefficient of brightness of the depth of the sea ρ (16) was determined empirically:

$$\alpha_{\downarrow}=0.068n,$$

where

$$n=\frac{\rho\,(549\,\mathrm{nm})}{\rho\,(483\,\mathrm{nm})}. \tag{45}$$

The correlation coefficient turns out to be equal to 0.94. In the same paper [11] the relationships between coefficients ρ and R was found:

$$\rho=0.28R. \tag{46}$$

In view of the fact that the index of absorption χ, which is one of the most important primary hydrooptical characteristics, applies to a number of values that are extremely difficult to measure directly, it is very interesting to discuss the relationships that make it possible to find index χ through other optical characteristics or parameters of the light field in the sea. Determination of χ from the simple relationship (24)

$$\chi = \varepsilon - \sigma,$$

as a rule leads to very great errors, since it is calculated on the basis of the difference between values that are very similar in magnitude and were also measured with low accuracy.

Determination of the index of absorption on the basis of the parameters of the light field follows from the equation for the divergence of the light vector of Gershun [4], which in the case of a plane-parallel medium will assume the form /20

$$\frac{\partial E(z)}{\partial z}=-\varkappa(z)\,E(z), \tag{47}$$

where E(z) is the flux density of the radiation as before (3).

On the basis of equation (47), V. N. Pelevin [15] obtained the following expression for the index of absorption:

$$\varkappa(z)=\alpha_{\downarrow}(z)\left[1-R(z)+\frac{R'(z)}{\alpha_{\downarrow}(z)}\right]\frac{E_{\downarrow}(z)}{E(z)}, \tag{48}$$

which, without any kind of assumptions, gives the exact relationship between index χ and the easily measured values $\alpha_{\downarrow}$, $R_{\downarrow}$, $E_{\downarrow}$ and E. For the surface layers of the sea (at optical depth $\tau \leqslant 2.5$-3.5) an approximate formula is obtained which is suitable for calculating χ in sunny weather with the height of the Sun $h_{\odot}$ no less than 15-20°:

$$\varkappa(z)=\alpha_{\downarrow}(z)\frac{1-R(z)+\frac{R'(z)}{\alpha_{\downarrow}(z)}}{\sec h_{\odot}+2.4R(z)}. \tag{49}$$

It should be noted that the value R changes very insignificantly with depth and the derivative R'(z) is very close to zero.

Solving (32) for $\chi(z)$ and noting the determination of the effective index of absorption (19), we will have

$$\varkappa(z)=\frac{\alpha_{\downarrow}(z)-R(z)\alpha_{\uparrow}(z)}{g_{\downarrow}(z)+R(z)g_{\uparrow}(z)}. \tag{50}$$

Oceanology has made very extensive use of measurements of the so-called "transparency" of the sea by means of a standard white disc (Sekki disc). Although the measured value by no means corresponds to the physical concept of transparency ($\Theta = e^{-\varepsilon}$), these measurements are definitely valuable. This is due on the one hand to the extreme simplicity of the measurement method and the directness of the results obtained, and on the other hand to the fact that there is a simple relationship between the depth of visibility of the disc and the optical characteristics which follows from the strict theory:

$$z_d=\frac{\log\left(\frac{r_d}{R}-1\right)+1.7}{0.43(\varepsilon+\alpha_{\downarrow})}, \tag{51}$$

where r_d is a coefficient of reflection of the disc.

We mentioned earlier that at great optical depths a certain stationary distribution of brightness as a function of angles has been established for the scattering of radiation. Attenuation of light with depth continues to obey the exponential law, but it proceeds more slowly than in the surface layers of the sea. Z. A. Ambartsunyan, as early as 1942, wrote: "...For diffuse radiation, there is a certain coefficient of attenuation which differs from the coefficient of attenuation of direct light" [16, p. 99]. The existence of a stationary distribution of brightness in the deep layers of a scattering medium was proposed by O. D. Khovol'son [17]. The transition to the depth mode in the sea was theoretically established by V. V. Shuleykin [18]. The experimental work of L. Whitney [19] confirmed the stationary distribution of brightness in the sea at considerable optical depth.

On achieving constancy of the relative distribution of brightness by angles the hydrooptical characteristics reach their limiting values:

$$\lim_{z \to \infty} \alpha_{\downarrow\uparrow}(z) = \gamma; \quad \lim_{z \to \infty} \chi(z) = \gamma. \tag{52}$$

Since in the depths of the sea it is only the scattered radiation which is propagated, instead of using the effective characteristics χ_{ef}, σ_{ef} and ε_{ef}, we must use the indices a, b and c used above, so that c has values of small $c = \gamma$.

The limiting values of the coefficients of angular distribution $g_{\downarrow}(z)$, $g_{\uparrow}(z)$ and $\bar{g}(z)$ will still be values that are independent of depth:

$$\lim_{z \to \infty} g_{\downarrow}(z) = g_{\downarrow}; \quad \lim_{z \to \infty} g_{\uparrow}(z) = g_{\uparrow}; \quad \lim_{z \to \infty} \bar{g}(z) = \bar{g}. \tag{53}$$

The coefficient of diffuse reflection of the sea R(z) also becomes constant with depth and, as follows from equation (44),

$$\lim_{z \to \infty} R(z) = \frac{c - a}{c + a}. \tag{54}$$

The relationship that exists under conditions of the deep mode between the indices of attenuation and absorption is presented in the work of G. V. Rosenberg [20]

(55)

For the same conditions, A. A. Gershun [4] presents the approximate relationship

$$\approx [illegible] \tag{56}$$

In [21], the approximate equation /22

$$\varkappa = 0.8\gamma \tag{57}$$

is given and it is noted that its accuracy does not exceed 10%.

In problems involving the optics of the sea, considerable interest attaches to the value of the relationship of index of attenuation under conditions of the deep mode to one of the basic primary hydrooptical characteristics, the index of attenuation

$$k = \frac{\gamma}{\varepsilon}. \tag{58}$$

It is precisely this parameter, for example, that it used in equations obtained by V. A. Ambartsumyan [16] for the case of bilateral infinity.

Substituting the value γ from (55) in (58)

$$k = \frac{\bar{g}\varkappa}{\varepsilon}$$

and taking into account the fact that it follows directly from a determination of the photon survival parameter (26) that

$$\frac{\varkappa}{\varepsilon} = 1 - \Lambda,$$

we will have

$$k = \bar{g}(1 - \Lambda). \tag{59}$$

For parameter k in [22] the following relationship is given:

$$k = \sqrt{0.23(1 - \Lambda)}. \tag{60}$$

For the conditions of the depth mode G. A. Gamburtsev [3] finds the solution to the system of equations of the type (39) in the form

$$R = \frac{a + b - \sqrt{a^2 + 2ab}}{b}.$$

This equation is easily converted to the following:

$$R = \frac{\frac{b}{a}}{1 + \frac{b}{a} + \sqrt{1 + 2\frac{b}{a}}}. \tag{61}$$

The latter expression shows once again the diffuse reflection of the sea is a function solely of the relationship between back scattering and true absorption. It has been established on the basis of a great deal of experimental data that the diffuse reflection does not change significantly with depth.

In the preceding text, we have listed a number of relationships that link the diffuse reflection of the sea with many hydrooptical characteristics. If we recall that the coefficient R is determined completely by the angular distribution of the radiation (33), there is every reason to assume that an analysis of the relationships presented can serve as a good tool for investigating the structure of the light field in the sea under various concrete conditions. This analysis can also be valuable for solving the reverse problem — the finding of the hydrooptical characteristics on the basis of the parameters of the light field, most of which can be measured very simply. /23

Let us use as the sake of an example some of the data that make it possible to put together an idea of the order of magnitude and the nature of their changes with depth for the indices of vertical attenuation $\alpha_{\downarrow}$ and $\alpha_{\uparrow}$ and absorption χ, the coefficient of diffuse reflection of the sea R and the coefficients of angular distribution of radiation $g_{\downarrow}$ and $g_{\uparrow}$. The values given in Table 3 for the characteristics just mentioned were given by R. Preisendofer [7] on the basis of data of J. Tyler [13], which he obtained at lake Pend Oreille (Idaho, USA), whose water, as we know, is known to be very similar in its properties to pure sea water and is distinguished by a homogeneity with depth. The observations were conducted during sunny weather ($h_{\odot} \approx 50°$), the measurements were conducted in the blue region of the spectrum (λ = 480 nm). The values of the index of absorption were calculated on the basis of the divergence of the light vector. The transparency of the water was characterized by the index of attenuation $\varepsilon = 0.17$ m^{-1}. The values of all the indices are represented as logarithms to the base 10.

The relationships stated in the present article between various hyrooptical characteristics are by no means exhausted, but they do make it possible to consider the basic interactions that take place between them and their dependence upon the parameters of the light field in the sea. /24

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TABLE 3.

Degrees, m	$\varepsilon_{\downarrow}$	$\varepsilon_{\uparrow}$	$\alpha_{\downarrow}$	$\alpha_{\uparrow}$	τ	R
4.2	1.25	2.70	0.059	0.057		0.022
7.3			0.069	0.067		
10.4	1.29	2.72	0.080	0.078	0.052	0.018
13.5			0.079	0.078		
16.6	1.29	2.78	0.077	0.077	0.053	0.020
22.8			0.076	0.076		
29.0	1.31	2.78	0.074	0.074	0.053	0.023
35.1			0.073	0.073		
41.3	1.32	2.76	0.072	0.072	0.053	0.026
47.5			0.071	0.071		
53.7	1.31	2.76	0.060	0.071	0.050	0.023
59.9						

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